Non-negative & Convex Linear Combinations

DEFINITION

Let $\vec{w} = \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \cdots + \alpha_n\vec{v}_n$. The vector $\vec{w}$ is called a ***non-negative*** linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ if

$$\alpha_1, \alpha_2, \ldots, \alpha_n \geq 0.$$

The vector $\vec{w}$ is called a ***convex*** linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ if

$$\alpha_1, \alpha_2, \ldots, \alpha_n \geq 0 \qquad \text{and} \qquad \alpha_1 + \alpha_2 + \cdots + \alpha_n = 1.$$

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9 Let

$$\vec{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \vec{c} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \vec{d} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad \vec{e} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

9.1 Out of $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$, and $\vec{e}$, which vectors are

(a) linear combinations of $\vec{a}$ and $\vec{b}$?

(b) non-negative linear combinations of $\vec{a}$ and $\vec{b}$?

(c) convex linear combinations of $\vec{a}$ and $\vec{b}$?

9.2 If possible, find two vectors $\vec{u}$ and $\vec{v}$ so that

(a) $\vec{a}$ and $\vec{c}$ are non-negative linear combinations of $\vec{u}$ and $\vec{v}$ but $\vec{b}$ is not.

(b) $\vec{a}$ and $\vec{e}$ are non-negative linear combinations of $\vec{u}$ and $\vec{v}$.

(c) $\vec{a}$ and $\vec{b}$ are non-negative linear combinations of $\vec{u}$ and $\vec{v}$ but $\vec{d}$ is not.

(d) $\vec{a}$, $\vec{c}$, and $\vec{d}$ are convex linear combinations of $\vec{u}$ and $\vec{v}$.

Otherwise, explain why it's not possible.

10 Let L be the set of points $(x, y) \in \mathbb{R}^2$ such that $y = 2x + 1$.

10.1 Describe L using set-builder notation.

10.2 Draw L as a subset of $\mathbb{R}^2$.

10.3 Add the vectors $\vec{a} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{d} = \vec{b} - \vec{a}$ to your drawing.

10.4 Is $\vec{d} \in L$? Explain.

10.5 For which $t \in \mathbb{R}$ is it true that $\vec{a} + t\vec{d} \in L$? Explain using your picture.